Solutions

Math 1A Spring 2025 Quiz 6

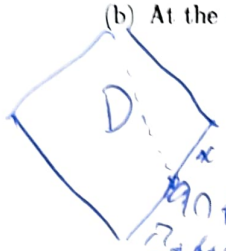
Name:

1. A baseball diamond is a square with side length 90 ft. A batter hits the ball and runs towards first base with a speed of 24 ft/s.

   (a) At what rate is his distance from second base decreasing when he is half way to first base?

   (b) At the same point, at what rate is his distance from third base increasing?

(A)



$D^2 = 90^2 + x^2$

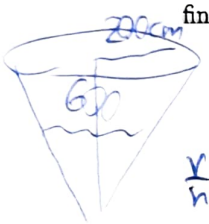
$2D\frac{dD}{dt} = 2x\frac{dx}{dt}$

$= 2(45)(-24)$

$\frac{dD}{dt} = \frac{2(45)(-24)}{2\left(\frac{90\sqrt{3}}{2}\right)} = -24/\sqrt{3}$ ft/sec.

B.) $+24/\sqrt{3}$

2. Water is leaking out of an inverted conical tank at a rate of 10,000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.



$\frac{dh}{dt} = +20$ cm/min

$\frac{r}{h} = \frac{2}{6}$ $\quad r = \frac{1}{3}h$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{27}h^3\pi$

$\frac{dV}{dt} = \frac{\pi}{27}\left(3h^2\frac{dh}{dt}\right) = \frac{\pi}{9}(200)^2(20)$

$\frac{dV}{dt} = R_{in} - R_{out}$

$= R_{in} - 10{,}000$

$R_{in} = \frac{\pi}{9}(200)^2(20) + 10{,}000$ cm^3/min

3. The minute hand of a watch clock is 8mm long and the hour hand is 4mm long.

   (a) Using the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos(C)$, find the distance between the tips of the two hands at one o'clock. Hint: What is the angle between the two hands at one o'clock?

   (b) How fast is the distance between the tips of the hands changing at one o'clock? (Hint: Consider the rate of change of the angle between the minute and hour hand. Differentiate the law of cosines function and use the answer from the previous part)

A.) $\pi/6$ $\rightarrow$ $c = \sqrt{8^2+4^2-2(8)(4)\frac{\sqrt{3}}{2}}$

B.) hr hand $\pi/6$ rad/hr $\quad \pi/6 - 2\pi = -11\pi/6$ rad/hr closure

min. 2π rad/hr.

$2c\frac{dc}{dt} = 2ab\sin(\theta)\frac{d\theta}{dt} \Rightarrow \frac{dc}{dt} = \frac{8(4)}{\sqrt{80-32\sqrt{3}}}\left(\frac{1}{2}\right)\left(-11\pi/6\right)$ mm/hr.